Exposing the non-collectivity in elliptic flow

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We show that backward-forward elliptic anisotropy correlation provides an experimentally accessible observable which distinguishes between collective and non-collective contributions to the observed elliptic anisotropy v_2 in relativistic heavy ion collisions. The measurement of this observable will reveal the momentum scale at which collective expansion seizes and where the elliptic anisotropy is dominated by (semi)-hard processes.

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Deconfined QCD matter at high energy density, the so-called Quark-Gluon Plasma (QGP), was a phase during the evolution of the early universe and it is now created and explored experimentally in relativistic heavy ion collisions. One major discovery by the experimental program [1] of the Relativistic Heavy Ion Collider (RHIC) is the large elliptic anisotropy[2], v_2, of observed particles' transverse momenta, which is consistent with predictions from ideal hydrodynamic expansion [3]. This led to the conjecture that the matter created in these collisions exhibits "perfect fluidity", i.e. minimal shear viscosity. First studies [4, 5, 6] indicate that the shear viscosity must be very small especially near T_c, and likely smaller than any known condensed matter substances and rather close to the conjectured universal lower bound $\frac{\eta}{s} \geq \frac{1}{4\pi}$ based on calculations utilizing the gauge/string duality [7].

The transverse momentum (p_t) dependence of the elliptic anisotropy, v_2, as depicted in Fig. 1 shows a rise at low p_t towards a maximum at $p_t \simeq 3\,\mathrm{GeV}$, and a constant value for large transverse momenta. At present the rise at low transverse momentum ($p_t \lesssim 1.5\,\mathrm{GeV}$) is thought to be due to *collective* hydrodynamic expansion, which translates the initial spatial anisotropy into a p_t anisotropy [8] over a wide region in pseudo-rapidity.

At very high p_t, on the other hand, v_2 is believed to result from the different attenuation of hard partons (jet quenching) in the asymmetrically distributed matter [9, 10, 11]. In this case the elliptic anisotropy, v_2, is *non-collective* in the sense that it is due to jet-like processes which are rather local in rapidity [12]. The transition from collective (hydrodynamic) to non-collective (jet-like) anisotropy is expected to take place around $p_t \simeq 2-4\,\mathrm{Gev}$, but the details are not well understood. In addition, one would expect that the attenuation of the jets should result in local, non-collective anisotropy also at lower p_t, since the debris from the jet-quenching process needs to go somewhere. If there is indeed a sizeable non-collective contribution to v_2 at low p_t, comparisons of (viscous) hydrodynamics with the data may lead to wrong conclusions about the viscosity of the produced matter in these collisions. Therefore, it would be desirable to have a direct measurement not only of the p_t scale at which the transition from collective to non-collective v_2 occurs, but also of the contribution of non-collective effects in the low p_t region, $p_t \lesssim 1.5\,\mathrm{GeV}$.

It is the purpose of this work to propose an observable which will distinguish between the collective and the non-collective contributions to the elliptic anisotropy. In this paper, for illustration purposes, we will identify the collective component with hydrodynamic flow and the non-collective one with attenuated jets. We note that there may be other mechanisms at work which generate similar collective and/or non-collective contributions to the elliptic anisotropy, and the proposed observable is not sensitive to any specific mechanism.

The key observation is the following: consider two rapidity bins, one forward, one backward, with a suitable separation (gap) in between. In a given event, the elliptic anisotropy of the collective component in both rapidity bins is highly correlated. The non-collective component, on the other hand, generates an elliptic anisotropy in either the forward bin or the backward bin, but never in both, resulting in very small backward-forward correlations. To illustrate this point, consider hydrodynamic expansion as an example for the collective component. In each event the elliptic anisotropy is aligned in azimuth and of comparable magnitude in the forward and backward bins. Contrast this with the elliptic anisotropy due to quenched jets, as an example for a non-collective component. At RHIC energies one rarely has more than one hard process per event. In addition, the anisotropy due to attenuated jets will be very local in rapidity and, therefore, will either contribute to the forward or to the backward bin but not to both. Consequently, these processes do not give rise to long-range forward-backward correlation of the elliptic anisotropy. At higher energies, such as the Large Hadron Collider (LHC) with many hard processes per event one has to choose a suitable rapidity gap, which is larger than typical jet cone extension but smaller than average jet-jet separation. After these general considerations, we will proceed to define the proposed observable and demonstrate its sensitivity to the degree of collectivity in contributions to v_2.

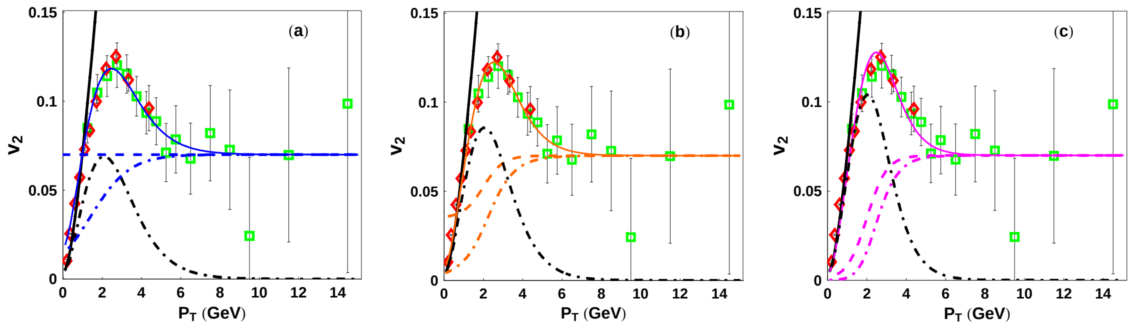

FIG. 1: Parametrization of v_2 data for AuAu 200 GeV, (00-20% centrality). The red diamonds are for negative charged hadrons [17] and the green boxes are for π_0 from PHENIX Run7 Preliminary [13, 18]. The blue/orange/magenta solid lines in (a)/(b)/(c) represent the combined parametrization, Eq.(9), respectively. In all three panels, the black solid lines show the collective-flow-only contribution while the colored long-dashed lines (extending to high p_t) show the jet-only contribution in Eq.(13). The dash-dotted lines show the weighted contributions $(1-g)\cdot < v_2^{\mathcal{F}} >$ (black) and $g\cdot < v_2^{\mathcal{J}} >$ (colored), respectively.

Let us start by recalling the definition of v_2:

$$\langle v_2(p_t)\rangle = \frac{\int_0^{2\pi} d\phi \cos(2\phi) \left\langle \frac{d^2N}{p_t dp_t d\phi}\right\rangle}{\int_0^{2\pi} d\phi \left\langle \frac{d^2N}{p_t dp_t d\phi}\right\rangle} \equiv \frac{\langle V_2(p_t)\rangle}{\left\langle \frac{dN}{p_t dp_t}\right\rangle} \quad (1)$$

In the above $\left\langle \frac{d^2N}{p_t dp_t d\phi}\right\rangle$ is the distribution of the event-averaged p_t-differential yield over the azimuthal angle ϕ which is defined with respect to the reaction plane. The numerator, denoted as V_2, may be referred to as the total elliptic anisotropy. Here we assume that the reaction plane is determined with high accuracy as it is achieved by current RHIC experiments. The particle yield at RHIC can be attributed to two main sources: the bulk matter which dominates low p_t regime and exhibits collective flow, and the (partially suppressed) hard jets dominant at high p_t. Both sources contribute to the total yield $\frac{d^2N}{p_t dp_t d\phi}$ and thus to the measured v_2.

The proposed new observable $\mathcal{C}_{FB}[p_T]$ is the correlation of the *total* $V_2(p_t)$ between forward (F) and backward (B) rapidity bins, specifically

$$\mathcal{C}_{FB}[p_T] \equiv \frac{< V_2^F \cdot V_2^B >}{< V_2^F > \cdot < V_2^B >} \quad (2)$$

where the total elliptic anisotropy in the forward, V_2^F, and backward, V_2^B, are defined as

$$\left\langle V_2^{F/B}(p_t)\right\rangle = \int_0^{2\pi} d\phi \cos(2\phi) \left\langle \frac{dN^{F/B}}{d\phi}\right\rangle, \quad (3)$$

$$\left\langle V_2^F \cdot V_2^B \right\rangle (p_t) = \left\langle \int_0^{2\pi} d\phi \cos(2\phi) \frac{dN^F}{d\phi} \cdot \int_0^{2\pi} d\phi' \cos(2\phi') \frac{dN^B}{d\phi'} \right\rangle \quad (4)$$

Here we denote by $\frac{dN^{F/B}}{d\phi} \equiv \frac{dN^{F/B}}{p_t dp_t d\phi} \cdot p_t \cdot \Delta p_t$ the ϕ-differential particle yield for a given p_t interval in the F/B bins, respectively. The forward/backward bins $[\pm y_{min}, \pm y_{max}]$ should be chosen such that the gap $(-y_{min}, y_{min})$ is sufficiently wide to prevent a jet from contributing to both simultaneously while still keeping both bins within the "flat plateau" near mid-rapidity.

The total yield, $\frac{dN^{F+B}}{d\phi} = \frac{dN^F}{d\phi} + \frac{dN^B}{d\phi}$, can be decomposed into two components[10], the hydrodynamic flow yield and the jet-related yield, respectively:

$$\frac{dN^{F+B}}{d\phi} = \left.\frac{dN^{\mathcal{F}}}{d\phi}\right|_{F+B} + \left.\frac{dN^{\mathcal{J}}}{d\phi}\right|_{F+B} \quad (5)$$

(In the following we drop the "$F+B$" subscript.)

To quantify the main idea, we express the yields in the F/B bins in each event as:

$$\frac{dN^F}{d\phi} = (\frac{1}{2} + \eta^F) \cdot \left\langle \frac{dN^{\mathcal{F}}}{d\phi}\right\rangle + \xi \cdot \frac{dN^{\mathcal{J}}}{d\phi} \quad (6)$$

$$\frac{dN^B}{d\phi} = (\frac{1}{2} + \eta^B) \cdot \left\langle \frac{dN^{\mathcal{F}}}{d\phi}\right\rangle + (1-\xi) \cdot \frac{dN^{\mathcal{J}}}{d\phi} \quad (7)$$

In the above we have introduced three random variables to schematically describe the fluctuations from event to event. $\eta^{F/B}$ represent (independent) random deviations from the average hydro flow yield in F/B bins, and satisfy $< \eta^{F/B} >= 0, < \eta^F \cdot \eta^B >= 0$. The variable ξ assumes values of either 0 or 1 in each event with equal probability, i.e. $< \xi >=< 1-\xi >= 1/2$ while $\xi(1-\xi) = 0$. The physical idea is that in each event there is at most one high p_t hadron cluster contributing to the final observed anisotropy V_2 [13] either in the forward or the backward rapidity bin. Calculations of dijet production at RHIC energy show a rapid decrease of cross section with increasing rapidity separation[14]. Also with a hard trigger in one bin, the back jet is strongly degraded in a heavy ion collision, inducing negligible contribution to v_2. In contrast, the hydrodynamic flow (ideal or viscous) contribution to the *anisotropy* is about equally split between F/B. The so-called anti-flow[15] gives rise to correlations between rapidity y and in-plane p_x. Its magnitude, v_1,

at RHIC energies is at most a few percent[2, 15] and induces a correction to v_2 of the order $\delta v_2 = v_1^2 < 1\%$. Furthermore it contributes equally to v_2 in both the F/B bins due to its (anti-)symmetry with respect to $y = 0$.

Next we introduce an interpolation function:

$$g(p_t) = \frac{\int_0^{2\pi} d\phi \left\langle \frac{dN^{\mathcal{J}}}{d\phi} \right\rangle}{\int_0^{2\pi} d\phi \left\langle \frac{dN^{\mathcal{F}}}{d\phi} \right\rangle + \int_0^{2\pi} d\phi \left\langle \frac{dN^{\mathcal{J}}}{d\phi} \right\rangle} \tag{8}$$

with $g(p_t)$ and $1 - g(p_t)$ giving the relative weight of the jet (non-collective) and hydro flow (collective) contribution to the total, $F + B$, yield, respectively. Given this function we express the observed $< v_2 >$ in the $F + B$ bins via (1) in terms of the flow and jet contributions:

$$\begin{aligned} < v_2(p_t) >&= \frac{\int_0^{2\pi} d\phi \cos(2\phi) \left[\left\langle \frac{dN^{\mathcal{F}}}{d\phi} \right\rangle + \left\langle \frac{dN^{\mathcal{J}}}{d\phi} \right\rangle \right]}{\int_0^{2\pi} d\phi \left[\left\langle \frac{dN^{\mathcal{F}}}{d\phi} \right\rangle + \left\langle \frac{dN^{\mathcal{J}}}{d\phi} \right\rangle \right]} \\ &= [1 - g(p_t)] \cdot < v_2^{\mathcal{F}} > + g(p_t) \cdot < v_2^{\mathcal{J}} > \end{aligned} \tag{9}$$

with $< v_2^{\mathcal{F}/J} >$ defined as in (1) with the corresponding hydro-flow-only/jet-only yields. Combining (3,4,6,7,8,9) and using the formalism of [16], we find for Eq.(2):

$$\mathcal{C}_{FB} = \frac{(1-g)^2 < v_2^{\mathcal{F}} >^2 + 2g\,(1-g) < v_2^{\mathcal{F}} >< v_2^{\mathcal{J}} >}{\left[(1-g) < v_2^{\mathcal{F}} > + g < v_2^{\mathcal{J}>} \right]^2} \tag{10}$$

A distinct feature is that as $g \to 0$ (hydro dominance) $\mathcal{C}_{FB} \to 1$ while $\mathcal{C}_{FB} \to 0$ in the other limit, $g \to 1$ (jet dominance). Thus, the correlation $\mathcal{C}_{FB}[p_t]$ distinguishes between collective and non-collective contributions and exposes the transition from the flow to the jet regime.

In order to demonstrate the sensitivity of $\mathcal{C}_{FB}$ we study three different scenarios within a simple model. All the scenarios employ the same blast-wave model for the collective flow contribution, but they differ in the p_t-dependence of the jet-only elliptic anisotropy, $\langle v_2^{\mathcal{J}}(p_t) \rangle$, which is not known for small $p_t \lesssim 3\,\mathrm{GeV}$. The relative strength, $g(p_t)$, of flow and jet contributions is obtained by fitting the v_2 data of the PHENIX collaboration [13, 17, 18] for AuAu $0 - 20\%$ centrality class at $\sqrt{s} = 200\,\mathrm{GeV}$ (see Fig.1). Scenario (a) assumes a constant $\langle v_2^{\mathcal{J}}(p_t) \rangle$ (see Fig.1(a)) . Scenarios (b) (Fig.1(b)) and (c) (Fig.1(c)) assume a constant $\langle v_2^{\mathcal{J}}(p_t) \rangle$ for large p_t which drops to half its value at $p_t = 0$ for scenario (b) and to zero for scenario (c). For both scenarios the momentum scale at which $\langle v_2^{\mathcal{J}}(p_t) \rangle$ changes towards the values at low p_t is $p_t = 2\,\mathrm{GeV}$.

The blast-wave model for the collective hydro flow contribution we adopt from ref. [19]. By parameterizing the flow velocity field at freeze-out as in [19] we have,

$$\left\langle \frac{dN^{\mathcal{F}}}{d\phi} \right\rangle \propto \frac{1}{(2\pi)^3} \int \frac{p^\mu d\sigma_\mu}{e^{p^\mu u_\mu / T_o} - 1} \tag{11}$$

The elliptic anisotropy is intrinsically built in the flow field $u^\mu(x^\mu)$: in $(\tau, \eta_s, r, \phi_s)$ coordinates $u^r = \frac{r}{R} u_o [1 + u_2 cos(2\phi_s)]\, \Theta(R_o - r)$, $u^\tau = \sqrt{1 + (u^r)^2}$, $u^{\phi_s} = u^{\eta_s} = 0$. The blast-wave parameters are chosen as $m_\pi = 140\,\mathrm{MeV}$, $T_o = 170\,\mathrm{MeV}$, $R_o = 10\,\mathrm{fm}$, $\tau_o = 7\,\mathrm{fm}$, $u_o = 0.7$, and $u_2 = 0.06$ which approximately reproduces the yield at low p_t [13]. Given this model we calculate the flow-only $< v_2^{\mathcal{F}} >$ via (1) using the yield in (11) (black solid lines in Fig.1(a,b,c)). We also show the weighted flow contribution $(1 - g) \cdot < v_2^{\mathcal{F}} >$ as the black dash-dot lines.

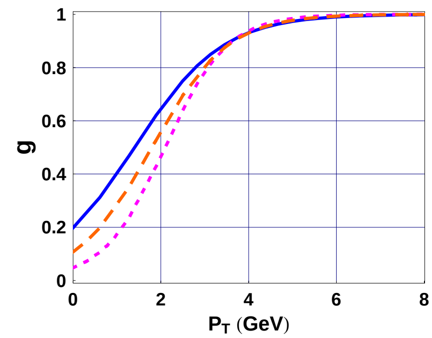

FIG. 2: The interpolation $g(p_t)$, Eq.(14), for scenarios (a) (solid line), (b) (long-dashed line), and (c) (short-dashed line).

In order to model the anisotropic jet attenuation we write the jet-related yield as

$$\left\langle \frac{dN^{\mathcal{J}}}{d\phi} \right\rangle \propto \left[1 + 2 \left\langle v_2^{\mathcal{J}} \right\rangle (p_t) \cdot \cos(2\phi) \right] \tag{12}$$

where $< v_2^{\mathcal{J}} > (p_t)$ parameterizes the resulting azimuthal anisotropy. While the experimental data indicate $< v_2^{\mathcal{J}} > (p_t)$ to be sizeable (7%) and rather *constant* for $p_t > 6\,\mathrm{GeV}$ [18], it is not clear how $v_2^{\mathcal{J}}$ behaves at lower p_t. To explore this we study the three scenarios described above which we parameterize as follows:

$$< v_2^{\mathcal{J}} >= 0.07 \left[\alpha + (1 - \alpha) \tanh(p_t - 2\,\mathrm{GeV}) \right] \tag{13}$$

The three scenarios (a), (b), and (c) correspond to $\alpha = 1$, $\alpha = \frac{3}{4}$, and $\alpha = \frac{1}{2}$, with the respective $< v_2^{\mathcal{J}} > (p_t)$ plotted in Fig.1 as colored long-dashed lines.

Finally, we parameterize the interpolation (8) as:

$$g(p_t) = \left[1 + \tanh[(p_t - P_C)/P_W] \right] / 2 \tag{14}$$

The parameters P_C, P_W (in units GeV) are obtained by χ^2 fitting of Eq.(9) to the data for v_2. The best choices for the three scenarios are: (a) $P_C = 1.4$, $P_W = 2$; (b) $P_C = 2.4$, $P_W = 1.8$; (c) $P_C = 2.4$, $P_W = 1.8$. In all cases a reasonable model description of the v_2 data is established over the whole p_t range, as can be seen in Fig.1. The resulting interpolation $g(p_t)$ are plotted in Fig.2. At low to intermediate p_t they differ considerably in the magnitude of the non-collective component. Of course the plotted curves are only examples for what may happen. To quantify $g(p_t)$ more experimental information will be required. We note, that an admixture of

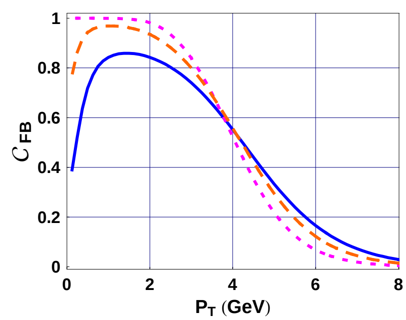

FIG. 3: The correlation $\mathcal{C}_{FB}(p_t)$ for scenarios (a) (solid line), (b) (long-dashed line), and (c) (short-dashed line).

a smaller jet-v_2 seems a plausible alternative to viscous corrections for a reduced v_2 at low to intermediate p_t.

Given the above parametrization we can calculate the proposed correlation $\mathcal{C}_{FB}$ using (10). The results are shown in Fig.3. While all three cases produce similar v_2, they are readily distinguishable by $\mathcal{C}_{FB}$ in the low to intermediate p_t region. The non-monotonic structure seen for scenarios (a) and (b) is due to interplay between a *non-vanishing* jet contribution and rapidly rising hydro flow contribution to v_2 at low p_t, which is absent in case (c). In addition, the deviation $\mathcal{C}_{FB}$ from unity at low p_t provides a measure for the non-collective contribution to v_2 in this region and their growth with increasing p_t. While the curves in Fig.3 depend on the specific parametrization, they demonstrate the sensitivity of $\mathcal{C}_{FB}$ to the non-collectivity which may hide in v_2.

In summary, we have proposed to use backward-forward correlations of the elliptic anisotropy to distinguish the collective (flow) from non-collective (jet) contributions to v_2. Using a two-component parametrization, we have studied the sensitivity of this observable to the degree of non-collectivity. We have further demonstrated that this observable is capable of exposing the transition from collective flow to jets with increasing p_t.

Concerning an actual measurement, one issue is that even though the anisotropy of the hydro flow field is perfectly aligned in the F/B bins, the actual v_2 of produced particles in the two bins may deviate (both in magnitude and in orientation) from the supposed "v_2" due to statistical fluctuation (see e.g. [20]). This will reduce the correlation from unity even for purely hydro flow, with the effect scaling as $1/\sqrt{N_{bin}}$. The other issue relates to the low yield in each event from intermediate to high p_t which may cause the correlation to vanish trivially. These problems may be partially cured by selecting proper p_t bin size and/or event trigger. Also those effects are small at low p_t region, where the particle abundance is large.

To conclude, a measurement of $\mathcal{C}_{FB}$ will reveal the magnitude of the non-collective contribution to v_2 and the transition pattern of v_2 between collective and non-collective behavior. The centrality dependence of this transition pattern would be of interest, as the flow and jet components scale differently with centrality. In addition, these measurements will help to understand the origin of the decreasing v_2 at intermediate p_t and constrain viscous corrections to the hydrodynamic evolution.

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